Derivation of Kepler's Third Law

It would be nice to predict the length of the relative period of a planet just from its relative distance to the sun, as compared to earth. The reason the ratio of periods of two different planets is not simply directly proportional to the ratio of orbital radii is that planets move faster in their orbits when they are closer to the sun. This leads us to a derivation of Kepler's Third Law. In reality, the paths of planets are elliptical. But we will simplify matters and assume they are circular. This will still lead to a pretty good approximation.

A planet's centripetal force is balanced by the gravitational force between the planet and the sun:

$$\frac{m_p v_p^{\,2}}{r_p} = \frac{G m_p m_s}{r_p^{\,2}}$$

where m_p = mass of the planet

V_p = average velocity of the planet
r_p = planet's average distance from the sun
m_s = mass of the sun
G = gravitational constant

Simplifying the above, we obtain:

$$v_p^{\,2} = \frac{G m_s}{r_p}$$

The same applies to the earth, so we will rewrite the formula using the subscript *e* to signify earth

$$v_e^{\,2} = \frac{G m_s}{r_e}$$

If we take a ratio of the two formulas we obtain:

$$\frac{v_p^{\,2}}{v_e^{\,2}} = \frac{r_e}{r_p}$$

Simplifying we obtain:

$$\frac{v_p}{v_e} = \sqrt{\frac{r_e}{r_p}}$$

The faster an object is the longer its period. A planet's period or time that it takes to orbit the sun once is *inversely* proportional to its speed:

$$\frac{P_p}{P_e} \propto \sqrt{\frac{r_p}{r_e}}$$

Of course, to create an equation all we are missing is the fact that the periods will also lengthen in direct proportion to the length of the path.

$$\frac{P_p}{P_e} = \frac{2\pi r_p}{2\pi r_e}\sqrt{\frac{r_p}{r_e}}$$

Simplifying we obtain:

$$\frac{P_p}{P_e} = \left(\frac{r_p}{r_e}\right)^{\frac{3}{2}}$$

If we set P_e= 1 year, all we need to estimate the time of revolution for each planet is how much further they are from the sun when compared to earth.
In the chart below the above formula is used to predict the time of revolution and then we compare it to the actual measured period.

Planet	distance from sun(km)	relative distanc	predicted period(yr)	actual pd.(yr)
Mercury	57,910,000	0.38709893	0.241	0.241
Venus	108,200,000	0.723262032	0.62	0.67
Earth	149,600,000	1	1.0	1
Mars	227,940,000	1.523663102	1.88	1.88
Jupiter	778,330,000	5.202740642	11.87	11.86
Saturn	1,429,400,000	9.554812834	29.53	29.6
Uranus	2,870,990,000	19.19110963	84.07	84.00
Neptune	4,504,000,000	30.10695187	165.2	160.8